Electrostatic Potential and Capacitance

- Electrostatic force is a conservative force. Work done by an external force (equal and opposite to the electrostatic force) in bringing a charge q from a point R to a point P is VP – VR, which is the difference in potential energy of charge q between the final and initial points.
- Potential at a point is the work done per unit charge (by an external agency) in bringing a charge from infinity to that point. Potential at a point is arbitrary to within an additive constant, since it is the potential difference between two points which is physically significant. If potential at infinity is chosen to be zero; potential at a point with position vector **r** due to a point charge Q placed at the origin is given is given by

  $V(r) = Q/4\pi\varepsilon 0r$
- The electrostatic potential at a point with position vector **r** due to a point dipole of dipole moment **p** placed at the origin is

  $V(r) = p.r'/4\pi\varepsilon 0r^2$

The result is true also for a dipole (with charges –q and q separated by 2a) for r >>a.

- For a charge configuration q1, q2, ..., qn with position vectors **r**1, **r**2, ... **r**n, the potential at a point P is given by the superposition principle

  $V = (1/4\pi\varepsilon 0)[(q1/r_{1p}) + (q2/r_{2p})+\ldots+(qn/r_{np})]$

  where r_{1P} is the distance between q1 and P, as and so on.

- An equipotential surface is a surface over which potential has a constantvalue. For a point charge, concentric spheres centred at a location ofthe charge are equipotential surfaces. The electric field E at a point isperpendicular to the equipotential surface through the point. E is in thedirection of the steepest decrease of potential.
- Potential energy stored in a system of charges is the work done (by anexternal agency) in assembling the charges at their locations. Potentialenergy of two charges q1, q2 at r1, r2 is given by

  U = = (1/4πε0)q1q2/r12

  where r12 is distance between q1 and q2.
- The potential energy of a charge q in an external potential V(**r**) is qV(**r**).

  The potential energy of a dipole moment **p** in a uniform electric field **E** is –**p.E**.

- Electrostatics field **E** is zero in the interior of a conductor; just outside the surface of a charged conductor, **E** is normal to the surface given by $E = (\sigma / \varepsilon_0) n$

  where **n**ˆ is the unit vector along the outward normal to the surface and σ is the surface charge density. Charges in a conductor can reside only at its surface. Potential is constant within and on the surface of a conductor. In a cavity within a conductor (with no charges), the electric field is zero.
- A capacitor is a system of two conductors separated by an insulator. Its capacitance is defined by $C = Q/V$, where Q and –Q are the charges on the two conductors and V is the potential difference between them. C is determined purely geometrically, by the shapes, sizes and relative positions of the two conductors. The unit of capacitance is farad:,

  $1F = 1\ C\ V^{-1}$. For a parallel plate capacitor (with vacuum between the plates),

  $C = \varepsilon_0 A/d$

where A is the area of each plate and d the separation between them.

- If the medium between the plates of a capacitor is filled with an insulating substance (dielectric), the electric field due to the charged plates induces a net dipole moment in the dielectric. This effect, called polarisation, gives rise to a field in the opposite direction. The net electric field inside the dielectric and hence the potential difference between the plates is thus reduced. Consequently, the capacitance C increases from its value C0 when there is no medium (vacuum),

  $C = KC_0$ where K is the dielectric constant of the insulating substance.

- For capacitors in the series combination, the total capacitance C is given by
  $(1/C) = (1/C_1) + (1/C_2)....+(1/C_n)$

In the parallel combination, the total capacitance C is:

$C = C1 + C2 + C3 + ...$

where C1, C2, C3... are individual capacitances.

- The energy U stored in a capacitor of capacitance C, with charge Q and voltage V is

  $U = (1/2)QV = (1/2)CV^2 = Q^2/2C$

  The electric energy density (energy per unit volume) in a region with electric field is $(1/2)\varepsilon 0E^2$.

- A Van de Graff generator consists of a large spherical conducting shell (a few metre in diameter). By means of a moving belt and suitable brushes, charge is continuously transferred to the shell and potential difference of the order of several million volts is built up, which can be used for accelerating charged particles.

Sample Examples

- A comb run through one's dry hair attracts small bits of paper.Why?

  What happens if the hair is wet or if it is a rainy day? (Remember,a paper does not conduct electricity.)

  (b) Ordinary rubber is an insulator. But special rubber tyres ofaircraft are made slightly conducting. Why is this necessary?

  (c) Vehicles carrying inflammable materials usually have metallicropes touching the ground during motion. Why?

  (d) A bird perches on a bare high power line, and nothing happensto the bird. A man standing on the ground touches the same lineand gets a fatal shock. Why?

  Solution

  (a) This is because the comb gets charged by friction. The moleculesin the paper gets polarised by the charged comb, resulting in anet force of attraction. If the hair is wet, or if it is rainy day, frictionbetween hair and the comb reduces. The comb does not getcharged and thus it will not attract small bits of paper.

To enable them to conduct charge (produced by friction) to theground; as too much of static electricity accumulated may resultin spark and result in fire.

(c) Reason similar to (b).

(d) Current passes only when there is difference in potential.

- A molecule of a substance has a permanent electricdipole moment of magnitude 10–29 C m. A mole of this substance ispolarised (at low temperature) by applying a strong electrostatic fieldof magnitude 106 V m –1. The direction of the field is suddenly changedby an angle of 60º. Estimate the heat released by the substance inaligning its dipoles along the new direction of the field. For simplicity,assume 100% polarisation of the sample.

  Solution

  Here, dipole moment of each molecules = 10^{-29} C mAs 1 mole of the substance contains 6×10^{23} molecules,

  total dipole moment of all the molecules, $p = 6 \times 10^{23} \times 10^{-29}$ C m= 6×10^{-6}C m

  Initial potential energy, $U_i = -pE \cos\theta = -6\times10^{-6}\times10^{6} \cos 0° = -6$ J

  Final potential energy (when $\theta = 60°$), $U_f = -6 \times 10^{-6} \times 10^{6} \cos 60° = -3$ J

Change in potential energy = –3 J – (–6J) = 3 J

So, there is loss in potential energy. This must be the energy releasedby the substance in the form of heat in aligning its dipoles.